

The fact that there are similarities between our SIS Josephson effect mixing results and the results with the very different point contact RSJ mixers is a new discovery which we failed to note in our paper. It might be thought that the SIS and the RSJ point contact junction are not significantly different. Taur's comprehensive theoretical analysis of the RSJ mixer explains the high noise of point contact mixers [3]. However, this theory is based on differential equations for the current response of the RSJ: it does not begin to address the SIS diode we used which has a significance capacitance and no resistive shunt, among other differences.

The first remarkable feature shared by RSJ mixers and our SIS Josephson effect mixer is the presence of excess noise. Taur's theory shows that this is due to the nonlinear interaction of the Josephson current with the Johnson noise from the shunt resistor in the RSJ mixer. It is reasonable to expect a similar result for the SIS Josephson mixer but Taur's theory does not apply.

The second remarkable feature shared by RSJ mixers and our SIS Josephson effect mixer is the ease with which they are each saturated by thermal noise. Jablonski's statement that "traditional measurements of noise temperature are inappropriate" in some microwave devices using Josephson effects is one with which I agree strongly. In Fig. 10 and the text around it, we make the point that the hot/cold load technique can be inaccurate even in an SIS mixer which is operated in its more usual non-Josephson mode. In the Josephson mixing mode, results from hot/cold load measurements were useless because of nonlinear response.

Our paper reports direct measurements of signal, and direct measurements of noise when that signal is present. There is no possibility of error in a measurement of mixer sensitivity made this way. Any saturation, or nonlinear response to signal would be directly seen by our measurement. Even if our mixer noise and conversion gain are affected by broadband noise on the SIS, our methods measure them correctly. I consider this to be a major point of our paper.

I agree that magnetic suppression makes Josephson currents circulate within the SIS, it does not eliminate them. However, the experimental evidence from submillimeter wavelength mixers is clear that these circulating currents do not degrade SIS mixer performance [4], [5]. Neither, in my opinion, do I see evidence for Jablonski's concern in our paper, which reports lower noise when the currents are forced to circulate by magnetic suppression.

To conclude, there is new work to be done in Josephson mixing using SIS's that was not done with the point contact mixer work of the past. The SIS and the point contact junction have very different equations governing their dynamics, so it is reasonable to investigate SIS based Josephson mixers. With planar SIS's, complicated tuning structures can be fabricated integrally with the chip, so much greater freedom in circuit design is available now than was available with point contact junctions. Therefore, it is useful to revisit the topic of Josephson mixing. We are not alone in this opinion: Josephson mixing with resistively shunted SIS's is currently being pursued at Caltech [6]. Their theoretical work suggests much lower noise mixers with the SIS circuits than with the older point contact work.

## REFERENCES

- [1] Y. Taur and A. R. Kerr, "Low-noise Josephson mixers at 115 GHz using recyclable point contacts," *Appl. Phys. Lett.*, vol. 32, June 1, 1978, pp. 775-777.
- [2] T. Poorter, "Josephson heterodyne detection at high thermal background levels," *J. Appl. Phys.*, vol. 53, pp. 51-58, Jan. 1982.

- [3] Y. Taur, "Josephson-junction mixer analysis using frequency-conversion and noise-correlation matrices," *IEEE Trans. Electron Devices*, vol. ED-27, pp. 1921-1928, Oct. 1980.
- [4] T. H. Büttgenbach, H. G. LeDuc, P. D. Maker, and T. G. Phillips, "A Fixed Tuned Broadband Matching Structure for Submillimeter Astronomy," *IEEE Trans. Appl. Superconduct.*, vol. 2, Sept. 1992.
- [5] C. K. Walker, J. W. Kooi, M. Chan, H. G. LeDuc, P. L. Schaffer, J. E. Carlstrom, and T. G. Phillips, "A Low-Noise 492 GHz SIS waveguide receiver," *Int. J. IR and MM Waves*, vol. 13, pp. 785-798, June 1992.
- [6] R. Schoelkopf, T. G. Phillips and J. Zmuidzinas, "Noise in Josephson effect mixers and the RSJ model," in *Proc. Third Int. Symp. on Space Terahertz Technology*, Ann Arbor, MI, 1992.

## Comments on "An Analytic Algorithm for Unbalanced stripline Impedance"

E. Costamagna and A. Fanni

**Abstract**—Results obtained from numerical inversion of the Schwarz-Christoffel conformal transformation are utilized to discuss data derived from the subject paper and from the subsequent comments in [1].

In the above paper,<sup>1</sup> algorithms derived from conformal mapping were presented by Robrish to calculate the characteristic impedance of *unbalanced* (or *offset*) stripline in homogeneous dielectric. The allowed accuracy was checked by comparing data computed using the boundary element method. Then, alternative evaluation methods have been discussed by Canright [1], for the Robrish geometry and for structures derived from it to account for undercut.

In principle, all these methods are approximate, and Canright's are applicable to a limited range of dimensions. Therefore, a comparison is useful with impedance data calculated using the numerical inversion of the Schwarz-Christoffel conformal transformation (SCNI), which has already been proved [2], [3] an accurate and reliable general purpose tool.

In Table I, the data computed by Canright [1, Table I] using the Robrish formulas and his own equation (1) in [1] and Wheeler's [4] or Cohn's [5] techniques for balanced striplines are compared with impedances computed by SCNI. In the second column, SCNI was applied to the whole geometry, assuming a magnetic wall along the vertical line of symmetry. In the fourth column, SCNI was utilized to complement formula (1) in [1], computing his impedances  $Z_{01}$  and  $Z_{02}$  (see Fig. 1 in [1]). As expected, because the ratio  $h_1/b = 1/3$  is not very small, the different data are in good agreement, and SCNI values merely confirm the previous evaluations.

Increasing the striplines offset, (1) in [1] leads to larger errors, as shown in Table II for  $h_1/b = 1/5$ . Errors range from about 4% to 6%, corroborating Robrish's opinion in his reply to [1]. This  $h_1/b$  ratio is the limit for which Robrish checked his formulas for maximum errors of 2%. Beyond this limit, errors were expected to increase rapidly: Table III shows the impedance values for  $h_1/b = 1/10$  and errors rise to more than 11%.

Manuscript received June 26, 1992.

The authors are with the Istituto di Elettrotecnica, Università di Cagliari, Piazza D'Armi, 09123 Cagliari, Italy.  
IEEE Log Number 920413.

<sup>1</sup>P. Robrish, *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 1011-1016, Aug. 1990.

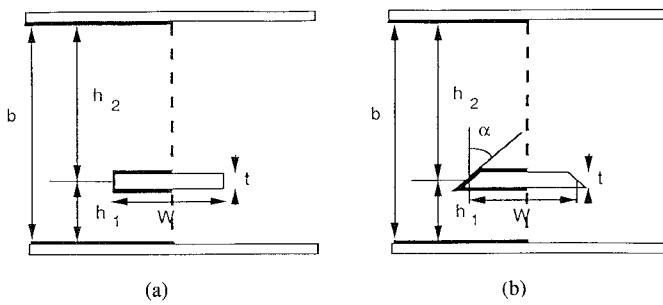


Fig. 1. Stripline structures: (a) without undercut; (b) with undercut. Thick lines show geometries transformed by SCNI.

TABLE I  
CHARACTERISTIC IMPEDANCE VALUES FOR THE STRUCTURE OF FIG. 1(a) WITH  
DIMENSIONS  $W = 5.00$  mil,  $t = 1.4$  mil,  $h_1/b = 1/3\epsilon_r = 4.8$

| b     | SCNI  | Robrish | [1, Eq. (1)] + | [1, Eq. (1)] + | [1, Eq. (1)] + |
|-------|-------|---------|----------------|----------------|----------------|
|       |       |         | SCNI           | Wheeler        | Cohn           |
| 13.86 | 40.20 | 40.00   | 40.75          | 40.72          | 40.30          |
| 20.27 | 50.11 | 50.00   | 50.87          | 50.88          | 50.37          |
| 24.33 | 54.79 | 55.00   | 55.82          | 55.83          | 54.44          |
| 35.05 | 64.79 | 65.00   | 65.82          | 65.81          | 64.68          |
| 50.59 | 74.70 | 75.00   | 75.87          | 75.85          | 74.86          |
| 60.60 | 79.67 | 80.00   | 80.91          | 80.88          | 79.93          |

TABLE II  
CHARACTERISTIC IMPEDANCE VALUES FOR THE STRUCTURE OF FIG. 1(a) WITH  
DIMENSIONS  $W = 5.0$  mil,  $t = 1.4$  mil,  $h_1/b = 1/5\epsilon_r = 4.8$

| b     | SCNI  | [1, Eq. (1)] + SCNI | % Difference |
|-------|-------|---------------------|--------------|
| 13.86 | 30.35 | 31.63               | 4.2          |
| 20.27 | 39.92 | 41.90               | 5.0          |
| 24.33 | 44.67 | 47.29               | 5.8          |
| 35.05 | 54.34 | 57.35               | 5.5          |
| 50.49 | 64.17 | 67.79               | 5.6          |
| 60.60 | 69.11 | 73.00               | 5.6          |

TABLE III  
CHARACTERISTIC IMPEDANCE VALUES FOR THE STRUCTURE OF FIG. 1(a) WITH  
DIMENSIONS  $W = 5.0$  mil,  $t = 1.4$  mil,  $h_1/b = 1/10\epsilon_r = 4.8$

| b     | SCNI  | [1, Eq. (1)] + SCNI | % Difference |
|-------|-------|---------------------|--------------|
| 13.86 | 15.07 | 15.85               | 5.9          |
| 20.27 | 23.85 | 25.64               | 7.5          |
| 24.33 | 28.26 | 30.65               | 8.5          |
| 35.05 | 37.41 | 41.15               | 10           |
| 50.49 | 46.92 | 52.06               | 11           |
| 60.60 | 51.77 | 57.59               | 11           |

Worth of mention is the accuracy achieved with the Canright method, based again on his formula (1) in [1], when accounting for undercut, (see Fig. 1(b)). Table IV shows the difference between approximate and exact values derived using SCNI for various undercut angles with  $h_1/b = 1/2$ .

For angles up to  $45^\circ$ , errors do not exceed 2.5%, covering with sufficient accuracy the range of practical computations. For sake of completeness, errors have been computed far beyond this range, for  $\alpha = 60^\circ$ , where they reach about 10%, and for  $\alpha = 80^\circ$ .

TABLE IV  
PERCENTAGE ERRORS OF EQ. (1) IN [1] FOR THE STRUCTURE OF FIG. 1(b) WITH  
DIMENSIONS  $W = 5.0$  mil,  $t = 1.4$  mil,  $h_1/b = 1/2\epsilon_r = 4.8$

| Offset angle $\alpha$ | 45° | 60° | 80° |
|-----------------------|-----|-----|-----|
| 13.86                 | 2.4 | 9.8 | 20  |
| 20.27                 | 2.1 | 8.7 | 18  |
| 24.33                 | 1.9 | 2.6 | 17  |
| 35.05                 | 1.6 | 7.4 | 15  |
| 50.49                 | 1.3 | 6.7 | 14  |
| 60.60                 | 1.7 | 6.8 | 13  |

Thus far, the vertical offset of the strip has been considered. Note that angular offset can be considered as well, by applying SCNI to geometries in which the vertical magnetic wall in Fig. 1 is replaced with curved boundaries, approximated by polygonal paths, matching the likely course of flux lines. Results of this approach have been discussed in [6] for thin striplines and in [3] for thick strips and for rectangular bars.

## REFERENCES

- [1] R. E. Canright, Jr., Comments on "An analytic algorithm for unbalanced stripline impedance," *IEEE Trans. Microwave Theory Tech.*, vol. 40, no. 1, pp. 177-179, Jan. 1992.
- [2] E. Costamagna and A. Fanni, "Characteristic impedances of coaxial structures of various cross section by conformal mapping," *IEEE Trans. Microwave Theory Tech.*, vol. 39, no. 6, pp. 1040-1043, June 1991.
- [3] —, "Analysis of rectangular coaxial structures by numerical inversion of the Schwarz-Christoffel transformation," *IEEE Trans. Magn.*, vol. MAG-28, no. 2, pp. 1454-1457, Mar. 1992.
- [4] H. A. Wheeler, "Transmission-line properties of a stripline between parallel planes," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, no. 11, pp. 866-876, Nov. 1978.
- [5] S. B. Cohn, "Problems in strip transmission lines," *IRE Trans. Microwave Theory Tech.*, vol. MTT-3, no. 2, pp. 119-126, Mar. 1955.
- [6] E. Costamagna, "TEM parameters of angular offset strip lines," *Alta Frequenza*, vol. LVII, no. 5, pp. 193-201, June 1988.

## Reply to Comments on "An Analytic Algorithm for Unbalanced Stripline Impedance"

P. Robrish

I'm pleased to see that Costamagna and Fanni have done some of the work which I had been critical of Canright for omitting, and it's always gratifying to have one's opinion confirmed independently. I found their comments on the accuracy of Canright's method for undercut lines interesting, but wish that they had extended their analysis to offset undercut lines. It would be useful to know how rapidly the errors vary with  $h_1/b$ . Since the undercut shape breaks one of the symmetries of the problem, one must consider values of  $h_1/b > 1/2$  as well as  $< 1/2$  in order to characterize the behavior adequately.

Manuscript received July 13, 1992.

The author is with Hewlett Packard Laboratories, P.O. Box 10350, Palo Alto, CA 94303.

IEEE Log Number 9204042.